



Algebra PoW Packet

Field Trip

April 20, 2009 • <http://mathforum.org/algpow/>

Welcome!

This packet contains a copy of the problem, the “answer check,” our solutions, teaching suggestions, and a problem-specific scoring rubric.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to “PoW Members” or use this URL to go to *algpow-teachers* directly: <http://mathforum.org/kb/forum.jsps?forumID=528> [Log in using your PoW username/password.]

The Problem

This problem is intended to have students write an equation or system of equations that winds up leading to solving a quadratic equation. While one of two quadratic equations is likely to result, both are factorable, and the hope is that students will solve by factoring rather than relying on their calculators to crank out the quadratic formula. As such, we are not including using the quadratic formula or completing the square in the Our Solution part of this document.

Here’s the problem:

Field Trip

The sixth grade is going on a field trip to the Science Museum, where they will spend the day and see a great 3-D IMAX movie. The museum offers a school group price of \$189, and the students will each pay an equal part of that total.

When rainy weather caused a baseball game to be rescheduled for the day of the field trip, 12 of the students had to miss the trip to play in the game. As a result, each student who went to the movie paid one dollar more than they would have if the baseball players had gone, too.

How many students went on the field trip, and how much did each one pay?



Answer Check

After students submit their solution, they can choose to “check” their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer), we provide hints and tips for those whose answer doesn’t agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation:

42 students went on the field trip. Be sure that you’ve also determined how much each one paid and clearly shown how you solved the problem.

If your answer **doesn't** match ours,

- * did you realize that dividing 189 by the number of students buying tickets gives the price per student?
- * did you try writing equations or expressions for both situations, with all the students or some of the students going on the trip?
- * if you had fractions in your equation, did you try multiplying both sides by the common denominator to clear the fractions?
- * did you check your arithmetic?

If any of those ideas help you, revise your answer. You can also leave a comment that tells what you changed and why you changed it. If you’re still stuck, leave a comment that tells where you’re having trouble.

If your answer **does** match ours,

- * have you used algebraic techniques to solve the problem?
- * did you explain how you came up with your equation(s) and why it works?
- * are there any hints that you would give another student?
- * are you confident that you could solve another problem like this successfully?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did - you might answer one or more of the questions above.

Our Solution

Method 1 - the cost per student

I started by choosing a variable:

Let n = the number of students in the sixth grade
 $n - 12$ = the number of sixth graders without the baseball players

I know that the trip costs \$189 and that the cost is shared equally among the students. That means the cost per student equals 189 divided by the number of students going. So originally, when the whole grade was going, the cost per student would have been $189/n$ dollars.

Once the baseball players couldn't go, only $n - 12$ students went on the trip, so the cost per student would be $189/(n - 12)$ dollars. I know that the second cost is \$1 more than the first one, so I wrote an equation expressing that:

$$\begin{aligned} (\text{cost with } n \text{ students}) + 1 &= (\text{cost with } n - 12 \text{ students}) \\ 189/n + 1 &= 189/(n - 12) \end{aligned}$$

I had two denominators with variables in them, so I multiplied both sides of the equation by $(n)(n - 12)$ to cancel those denominators. I wound up with a quadratic equation, so I set it equal to 0

$$\frac{(n)(n - 12)}{1} * \left[\frac{189}{(n)} + \frac{1}{1} \right] = \frac{189}{(n - 12)} * \frac{(n)(n - 12)}{1}$$

$$189(n - 12) + 1(n)(n - 12) = 189(n)$$

$$189n - 2268 + n^2 - 12n = 189n \quad (\text{the } 189n \text{ on each side cancels})$$

$$n^2 - 12n - 2268 = 0$$

I factored the left side by finding factors of 2268 that differed by 12 (42 and 54). Since the product of the factors is 0, one or both of them must be 0, so I solved each factor to see what value of n would make it 0.

$$n^2 - 12n - 2268 = 0$$

$$(n - 54)(n + 42) = 0$$

$$n - 54 = 0 \quad \text{or} \quad n + 42 = 0$$

$$n = 54 \quad \text{or} \quad n = -42$$

The total number of sixth graders can't be negative, so I rejected the -42. That means there are 54 sixth graders. I divided 189 by 54 and found that each student would have paid \$3.50 if they all went.

Without the twelve baseball players, only $54 - 12$ or 42 students went. I divided 189 by 42 and found that each student paid \$4.50. That checks since it's \$1 more than \$3.50, so the ticket price went up one dollar without the baseball players.

42 students went on the trip, and each paid \$4.50.

Method 2 - replacing the dollars

I started by choosing a variable:

Let n = the number of students in the sixth grade
 $n - 12$ = the number of sixth graders without the baseball players

The sixth grade needs to have \$189 to go on the trip. At the start, all n of the students would pay $189/n$ dollars each to split the cost equally. When the 12 baseball players couldn't go, their twelve shares of the \$189 were lost. That left the rest of the kids 12 times $189/n$ dollars short of 189.

In order to make up for that shortfall, the remaining $n - 12$ sixth graders each paid one dollar more, in addition to what they would have paid already. That means that the $(n-12)$ times (1) additional dollars they paid exactly replaced the 12 times $189/n$ dollars that were lost. So those two amounts must be equal, and I wrote an equation showing that:

$$(12)(189/n) = (n - 12)(1)$$

To get rid of the n in the denominator, I multiplied both sides of the equation by n. I also multiplied 12 times 189 and got 2268:

$$\begin{aligned} n \cdot [2286/n] &= [n - 12] \cdot n \\ 2268 &= n^2 - 12n \\ 0 &= n^2 - 12n - 2268 \end{aligned}$$

From here, this method finishes like Method 1 above.

Method 3 - a system of equations

I chose variables to represent the four unknown pieces of information in the problem:

$$\begin{aligned} \text{Let } n &= \text{the number of students in the sixth grade} \\ c &= \text{the cost in dollars per student for the whole grade} \\ n - 12 &= \text{the students who went on the trip without the players} \\ c + 1 &= \text{the cost in dollars per student without the players} \end{aligned}$$

I know that the total cost of the trip is \$189 and that equals the number of kids on the trip times how much each one pays. So I wrote two equations, one for the whole sixth grade going and one for after the baseball players couldn't go:

$$\begin{aligned} nc &= 189 \\ (n - 12)(c + 1) &= 189 \end{aligned}$$

I distributed out the second equation and then substituted 189 for nc from the first equation and simplified:

$$\begin{aligned} (n - 12)(c + 1) &= 189 \\ nc + n - 12c - 12 &= 189 \\ 189 + n - 12c - 12 &= 189 \\ n - 12c - 12 &= 0 \end{aligned}$$

Note: From here there are a variety of different ways to solve the system of equations, including which variable to solve for first. We'll show some of them, but others are certainly valid.

I solved my first equation for n and got $n = 189/c$, then substituted that into the new equation so it had only c in it:

$$\begin{aligned} n - 12c - 12 &= 0 \\ 189/c - 12c - 12 &= 0 \end{aligned}$$

I multiplied the whole equation by c to cancel out the c under the 189. I found I had a quadratic equation so I set it equal to 0, then divided everything by 3 since all the numbers are multiples of 3:

$$\begin{aligned} c \cdot [189/c - 12c - 12] &= c \cdot [0] \\ 189 - 12c^2 - 12c &= 0 \\ 0 &= 12c^2 + 12c - 189 \\ 0 &= 4c^2 + 4c - 63 \end{aligned}$$

I factored the right side and then solved each factor to see what value of c would make it 0 since their product has to be 0 and that only happens when at least one of the factors is 0:

$$\begin{aligned} 0 &= 4c^2 + 4c - 63 \\ 0 &= (2c - 7)(2c + 9) \\ 2c - 7 &= 0 \quad \text{or} \quad 2c + 9 = 0 \\ 2c &= 7 & 2c &= -9 \\ c &= 7/2 & c &= -9/2 \end{aligned}$$

It doesn't make sense for the cost per student to be negative, so I rejected the $-9/2$ answer. That means the cost for each student was $7/2$ dollars or \$3.50. Substituting that into the equation $nc = 189$ I solved for n:

$$\begin{aligned} nc &= 189 \\ 3.5n &= 189 \\ n &= 54 \end{aligned}$$

There are 54 sixth graders and the original plan was for them to pay \$3.50 each. When the 12 baseball players couldn't go, the $54 - 12$ or 42 remaining students paid $3.50 + 1$ or \$4.50 each. To check, I multiplied 42 times 4.50 to see if it made 189, and it did.

Some alternative solutions to the system

One alternative path to getting the same quadratic equation would be to take the original two equations and solve the second one for n , then substitute that expression in for n in the first equation:

$$\begin{aligned}nc &= 189 & \text{and} & & n - 12c - 12 &= 0 \\nc &= 189 & \text{and} & & n &= 12c + 12 \\c(12c + 12) &= 189 \\12c^2 + 12c &= 189 \\12c^2 + 12c - 189 &= 0\end{aligned}$$

Another alternative is solving for n instead of c originally, which results in the same quadratic equation we saw in Methods 1 and 2 above. Starting with the same two equations:

$$\begin{aligned}nc &= 189 \\n - 12c - 12 &= 0\end{aligned}$$

This time I solved the first equation for c and got $c = 189/n$, then substituted that expression for c in the second equation:

$$\begin{aligned}n - 12c - 12 &= 0 \\n - 12(189/n) - 12 &= 0\end{aligned}$$

I multiplied the whole equation by n to cancel out the n under the 189 and multiplied 12 times 189 to get 2268. I found I had a quadratic equation so I set it equal to 0:

$$\begin{aligned}n*[n - 2268/n - 12] &= [0]*n \\n^2 - 2268 - 12n &= 0 \\n^2 - 12n - 2268 &= 0\end{aligned}$$

From here this approach finishes the same way as Methods 1 and 2.

Teaching Suggestions

The first draft of this problem had whole number values for the amounts the students paid, and in our discussions of it some of us said that it was easier to use guess and check and just think about the factor pairs of the total cost (since the number of students would have to be whole numbers) than to do all the algebra. We changed the problem to make the dollar amounts non-whole, but it was an interesting discussion about the use of algebra that I'd like to share a bit of here.

It's true that many of the algebra problems we do, particularly in an Algebra I course (which is the target for the Algebra Problem of the Week), can be solved using logical guessing and checking. We're limited to making the problems not too challenging because students aren't equipped to handle them yet. So it's important to help kids see the value of algebra in solving more complex problems, even if more complex just means bigger or uglier numbers that make the guessing process more difficult. What if in this problem the total cost of the trip was \$720 and the change in per-student price when 15 kids missed the trip was \$1.60 instead of one dollar? Although the problem is identical, trying to solve that one by guessing and checking would be very tedious, while using algebraic steps should take about the same amount of time as the actual version.

As teachers, we want to help kids see that the thinking they use in guessing and checking is essentially the same thinking that's used in an algebraic solution, but that rather than trying a specific number we try a variable and the calculations we use to do the guessing and checking are what lead to the algebraic equation. Many students have not made that connection, so it's something to keep working on with them. You might pose a similar question to this problem with very small numbers where the solution is quickly evident, then have students try generalizing their guessing strategy by replacing the number they guessed with variables and variable expressions.

For the more proficient students who get to a quadratic equation in the problem, there is the opportunity to focus on any of the standard quadratic solving paths - factoring, completing the square, use of the quadratic formula, or graphing. It has seemed recently that more and more kids have a quadratic formula program in their calculator where they just enter a , b , and c and get the answer. Calculators are great tools, but I think it's important that kids really understand what the calculator is doing for them.

Finally, an interesting exploration with this problem is to pose the question, "How many kids would have to miss the trip for the amount each remaining student paid to double?" Some kids may see right away that it would require half the kids to miss the trip, but that result can also be determined algebraically fairly easily. Extending the thought, ask how many would need to miss the trip for the amount each remaining student paid to triple, quadruple, and so on. Ask them to find a function or

rule that expresses what fraction of the kids must miss the trip for any given multiple of the original per-student cost. There is a nice pattern in that question that can be easily expressed as a function and provides good practice in thinking about functions.

Scoring Rubric

The **problem-specific rubric** is something we write for every problem for use by those who are assessing student work. It spells out what we expect from students in three areas of problem solving and three areas of communication. The goal is to assess a student response within each category as it relates to the specific criteria for that category. This approach allows you to retrieve more targeted information on the students' areas of strength and weakness.

In the most general sense, *Interpretation* usually includes understanding the given information, including any diagram, attempting to answer all parts of all of the questions asked, and exhibiting understanding of any fundamental math concepts necessary to solve the problem. *Strategy* is usually then applying all of that knowledge in a systematic and mathematically sound way that doesn't rely on any lucky guesses. *Accuracy* simply refers to executing your strategy correctly. Note that even an incorrect strategy can be done accurately. *Completeness* is showing and explaining the thinking and work you did to reach your answer, and *Clarity* is presenting that explanation in a way that is easy for someone else to read and understand. *Reflection* includes such things as thinking about the reasonableness of your answer, checking it, tying the problem to past problems, and summarizing the key ideas in your solution.

A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on any problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

The problem-specific rubric is shown on the next page.

Happy Problem Solving!

We hope these packets are useful in helping you make the most of the AlgPoWs. Please let me know if you have ideas for making them more useful.

– RIZ

riz@mathforum.org

Algebra Problem of the Week Scoring Rubric for Field Trip

For each category, choose the level that *best describes* the student's work.

Problem Solving		Novice	Apprentice	Practitioner	Expert
Interpretation	shows understanding of only one or two of the concepts involved - see the Practitioner column	shows understanding of most but not all of the concepts in the Practitioner column (for example, understands that 12 fewer kids went than originally planned, but doesn't realize that the trip still costs a total of \$189)	shows understanding that: the \$189 cost is split evenly among those who go so 189 equals the number of kids times what each pays 12 kids fewer than the whole grade went on the trip the price per student went up \$1 when those 12 didn't go	not possible on this problem since there is no Extra question to show expert interpretation of the concepts involved	
Strategy	has few ideas that will lead them toward a successful solution	picks an incorrect strategy, or relies on luck to get the right answer uses Guess and Check, either for entire solution or to solve an equation	picks a sound strategy—success achieved through skill, not luck represents the problem using algebra, possibly by comparing ticket price expressions for the two cases or by writing a system of equations or by graphing work is accurate and contains no arithmetic mistakes	uses two separate strategies or an unusual or sophisticated strategy	
Accuracy	work contains many errors	work is mostly accurate, with a few errors		generally not possible - can't be more accurate than Practitioner	
Communication					
Completeness	has written very little that tells or shows how they found their answer	explains the steps used to find the answer but shows very few of the calculations and work OR shows the work but does not explain the thinking behind it does not define variable(s)	defines any variables that are used explains and shows all of the steps taken to solve the problem, including any equations, formulas and calculations used explains how those equations represent the facts and relationships in the problem and shows how they were solved	adds in useful extensions and further explanation of some of the ideas involved the additions are helpful, not just "I'll say more to get more credit"	
Clarity	explanation is very difficult to read and follow	explanation isn't entirely unclear, but would be hard for another student to follow explanation is long and is written entirely in one paragraph explanation contains many spelling and typing errors	explains all of the steps in such a way that another student would understand the work and thinking used in the solution makes an effort to check their formatting, spelling, and typing (a few errors are fine as long as they don't make it hard to read)	formats things exceptionally clearly answer is very readable and appealing	
Reflection	The items in the columns to the right are considered reflective. They could be in the solution or the comment left after viewing the Math Forum's answer. did nothing reflective	checked answer in some way (in addition to viewing the answer provided by the Math Forum) reflected on the reasonableness of their answer did one reflective thing	connected the problem to prior problems or experiences explained where they are stuck summarized the process they used did two reflective things	commented on and explained the ease or difficulty of the problem revised and improved their work	did three or more reflective things or did an exceptional job with two of them