



Pre-Algebra PoW Packet

Suli's Sense

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Welcome!

This packet contains a copy of the problem, the “answer check,” our solutions, teaching suggestions, and a problem-specific scoring rubric. The problem is new, so we have no student work to share.

This problem was adapted from a problem from the unit "Dealing with Data" (Section D: Histograms and the Mean) from Encyclopædia Britannica's: *Math in Context* series. View the student pages here on Britannica's site:

http://mathincontext.eb.com/teacher/data2/20_dwd_sb.pdf

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to “PoW Members” or use this URL to go to *prealgpow-teachers* directly: <http://mathforum.org/kb/forum.jsps?forumID=527> [Log in using your PoW username/password.]

The Problem

In *Suli's Sense*, students are asked to explain if Suli's result was a coincidence.

The text of the problem is included below. A print-friendly version is available from the “Print this Problem” link on the current PreAlgPoW problem page.

Suli's Sense

In the newspaper Suli found the mean high temperature in her town last week was 72° F. The high for each day was also listed. She decided to explore how the high temperature each day differed from the mean. When she added the differences, she got 0.

Question: Was this a coincidence? Explain how you know.



Adapted from Encyclopædia Britannica: *Math in Context*
Dealing with Data: Section D: Histograms and the Mean

Answer Check

It was **not** a coincidence that Suli got a sum of 0.

If your answer does **not** match ours,

- have you checked the Ask Dr. Math archive to remind yourself about how to find the mean? <http://mathforum.org/library/drmath/view/58326.html>
- did you try guessing a data set?
- did you use a table to help you organize your work?
- did you notice that when the daily high is below 72 the difference is negative?

If any of those ideas help you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- did you explain why you are confident that Suli's result is not unusual?
- and if you created a data set, did you explain how you know that you and Suli didn't just both get lucky?
- did you make any mistakes along the way? If so, how did you find them?

- what hints would you give another student trying to solve this problem?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

The main ideas in this problem are mean, temperature, and integers.

Method 1: Make up sample data sets

The question wants to know if it's a coincidence that Suli got 0 when she added up all of the differences in her data set, but it doesn't give the numbers in the data set. That means I can't just check her arithmetic and see if she's right.

If it's a coincidence than Suli just happened to pick a "good" data set, maybe I could try to make up my own data set and see what I get.

I know that the mean has to be 72. The easiest way for that to happen is if each day the high was 72. The differences would always be 0 because $72 - 72 = 0$. If Suli summed all those zeroes, her answer would also be zero.

But Suli's data set is probably more complicated, or she wouldn't be wondering.

What other data sets have a mean of 72? Well, 62 and 82 have a mean of 72, because it's right in the middle. And 52 and 92 do, too. And so do 42 and 102. The data set could be: 42, 52, 62, 72, 82, 92, 102.

Let me check the differences:

Temperature	42	52	62	72	82	92	102
Difference	30 less	20 less	10 less	0	10 more	20 more	30 more

$$-30 + -20 + -10 + 0 + 10 + 20 + 30 = 0.$$

So with that more complicated data set, the sum of the differences is still 0. It doesn't seem like a coincidence because I picked numbers that were the same distance away from 72 on both sides.

Let me try numbers that are more reasonable. What if the numbers were

$$69, 70, 71, 72, 73, 74, 75$$

72 is still the middle (and the mean), because

$$(69 + 70 + 71 + 72 + 73 + 74 + 75)/7 = 504/7 = 72$$

The differences are -3, -2, -1, 0, 1, 2, 3

$$\text{and } -3 + -2 + -1 + 0 + 1 + 2 + 3 = 0$$

I think that if you make a data set by using pairs of numbers that are the same distance from 72, you will always get that the sum of the differences is 0 and the mean is 72, so I don't think it's a coincidence.

Method 2: Reasoning from the sum of the numbers

I wonder how I could find a data set whose mean is 72 so I could see what the sum of the differences is. Then I might be able to tell if Suli's data set was a special coincidence or not.

I know if the mean is 72 some of the numbers have to be below 72 and some have to be above to balance it out. I also noticed that the sum needs to give you 72 when you divide by 7, so the sum needs to be $72 \times 7 = 504$.

That means I could make a data set that works by picking 6 temperatures randomly. I've picked 59, 60, 63, 65, 75, 80 and I've set the 7th number to 504 minus those 6 numbers.

$$504 - (59 + 60 + 63 + 65 + 75 + 80) = 102$$

So 59 + 60 + 63 + 65 + 75 + 80 + 102 would work as my data set.

What are the differences? I decided to always do daily temp – mean temp so that I would get negative numbers when the number was below the mean and positive numbers when it was above the mean.

$$\begin{aligned}
 59 - 72 &= -13 \\
 60 - 72 &= -12 \\
 63 - 72 &= -9 \\
 65 - 72 &= -7 \\
 75 - 72 &= 3 \\
 80 - 72 &= 8 \\
 102 - 72 &= 30
 \end{aligned}$$

$$\text{Sum: } -13 + -12 + -9 + -7 + 3 + 8 + 30 = 0$$

It worked with a totally made-up data set! I bet it's not a coincidence. I know the sum is always going to be 504. Let me see what happens if I change one of the numbers. If I lower 102 to be a more reasonable temperature, like 85, then the sum went down

$$\text{by } 102 - 85 = 17.$$

To keep the total the same, I need to raise the other numbers by a total of 17.

I'll give 5 to 59, 4 to 60, 3 to 63, 2 to 65, 1 to 75, and 2 to 80.

Data: 64, 64, 66, 67, 76, 82, 85.

Total: 504

Mean: 72

Differences:

$$\begin{aligned}
 64 - 72 &= -8 \text{ (difference went up by 5)} \\
 64 - 72 &= -8 \text{ (difference went up by 4)} \\
 66 - 72 &= -6 \text{ (difference went up by 3)} \\
 67 - 72 &= -5 \text{ (difference went up by 2)} \\
 76 - 72 &= 4 \text{ (difference went up by 1)} \\
 82 - 72 &= 10 \text{ (difference went up by 2)} \\
 85 - 72 &= 13 \text{ (difference went down by 17)}
 \end{aligned}$$

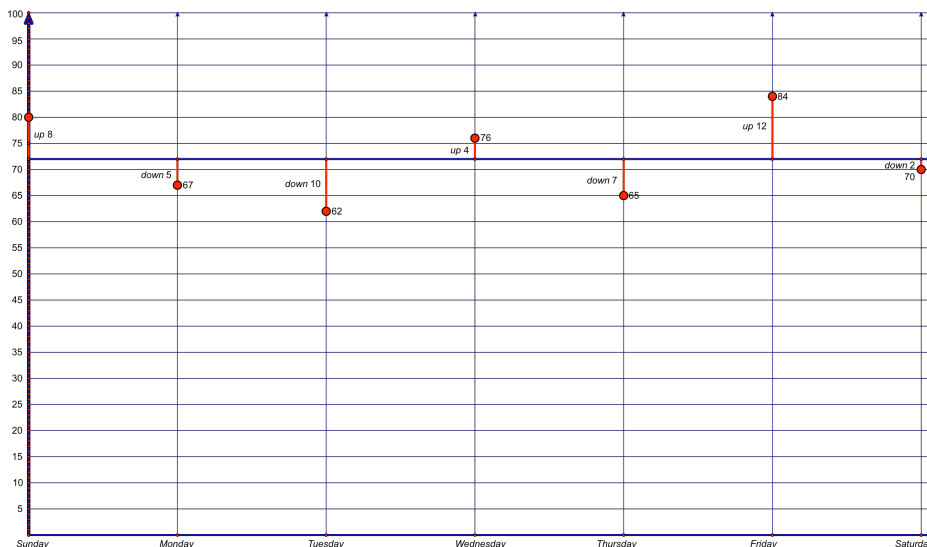
$$\text{And when I add them I get: } -8 + -8 + -6 + -5 + 4 + 10 + 13 = 0$$

The sum still balances out to 504 and the differences still balance out to 0. It is not a coincidence that Suli got 0 for the sum of the differences in her data set.

Method 3: Thinking Visually

I thought if I made a graph of Suli's data set, it might help me see if the differences are 0 for a reason, or it's just a coincidence. I don't know what numbers Suli had in her data set but I could make up a data set of my own using a graph. I guessed a data set to get started, and then adjusted it to make the mean be 72. I randomly guessed

80, 67, 62, 76, 65, 84, 70 and made this graph:



$$\text{Mean: } (80 + 67 + 62 + 76 + 65 + 84 + 70)/7 = 504/7 = 72.$$

Whoah! I guessed a data set that works exactly! Weird...

Now I'll check the differences:

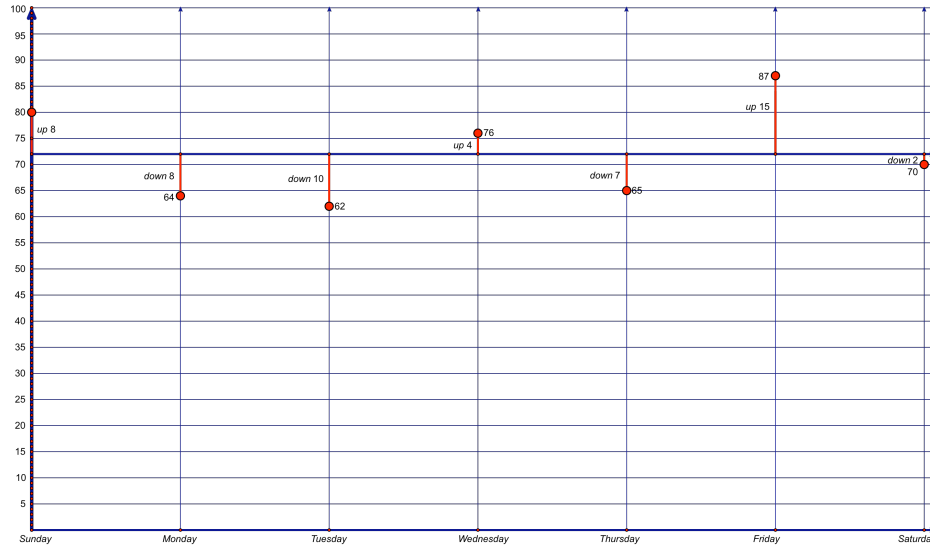
$$\text{up } 8 + \text{down } 5 + \text{down } 10 + \text{up } 4 + \text{down } 7 + \text{up } 12 + \text{down } 2.$$

I'll do positive for up and negative for down to do the adding.

$$8 - 5 - 10 + 4 - 7 + 12 - 2 = 0.$$

Wow, that's what Suli got too. Either I'm a really amazing guesser, or it's not a coincidence.

Let me see if I can adjust the data so the mean is still 72. I know that if I adjust some numbers up, I need to adjust other numbers down so the mean stays 72.



I adjusted Friday's temperature of 84 up to 87, so I adjusted Monday's 67 down to 64 to compensate. I checked that the mean was still 72.

$$\text{Mean: } (80 + 64 + 62 + 76 + 65 + 87 + 70)/7 = 504/7 = 72.$$

Yup. And I know that the sum of the differences didn't change either, because I made the Friday temperature three units higher than the mean, but I made Monday's temperature 3 units lower than the mean.

In order for the mean to be 72, the numbers have to compensate on either side. That means when you add up all the differences, you have to have the same amount above the mean as below the mean, which means the sum of the differences is 0 (if you make below the mean be negative numbers and above the mean be positive).

It is not a coincidence that Suli got 0 for the sum of the differences in her data set.

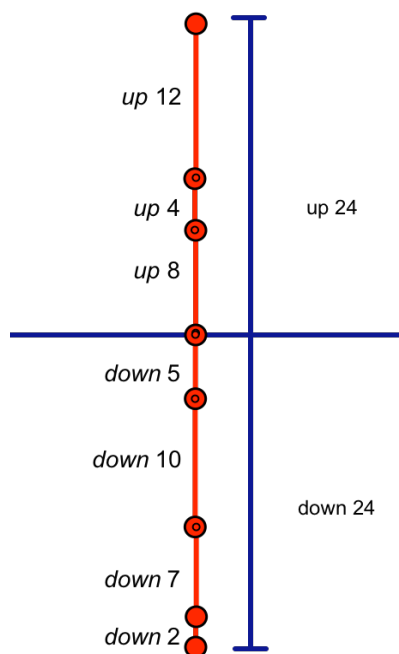
Method 4: Thinking of Pushing and Pulling

I don't know what numbers Suli had in her data set but I could make up a data set. I picked some numbers. I thought about the sum of the differences and they stayed the same. The mean is:

$$(79 + 66 + 61 + 76 + 65 + 87 + 70)/7 = 504/7 = 72.$$

I thought about what was happening. The mean stays the same as long as the sum of the differences stays the same, which makes sense because the amount the data pushes up above 72 is balanced out by the amount the data pulls down below 72.

Here is what I mean visually by pushing and pulling. In order for the mean to be 72, the numbers have to compensate on either side. That means when you add up all the differences, you have to have the same amount above the mean as below the mean, which means the sum of the differences is 0 (if you make below the mean be negative numbers and above the mean be positive).



It is not a coincidence that Suli got 0 for the sum of the differences in her data set.

Teaching Suggestions

One of the conversations we had in the office as we were thinking about how students might work this problem is if they only considered each day to have a high temperature of 72. Would we accept this data set as conclusive evidence? Would we expect pre-algebra level students to prove conclusively that Suli's answer was not a coincidence? How much "proof" would be enough at this level?

Resist the urge to give direct instructions on a specific approach. Ask students to paraphrase the problem to check on their understanding before they begin working on it. Ask questions that help them understand the language of the problem, visualize it, and discover patterns. Good questions help students clarify their thinking and give you useful information as well.

The questions in the Answer Check, above, might serve as good prompts to help students make progress. Encourage students to use a strategy that works for them. You can see from the various methods that we have thought to use for this problem that there are many ways to approach this problem. And, we may not have thought of them all!

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources:

<http://mathforum.org/prealgpow/puzzles/supportpage.ehtml?puzzle=417>

The Problem Solving and Communication Activity Series document for this problem contains ideas and activities to help students:

http://mathforum.org/pow/support/activityseries/prealgpow_psc.417.pdf

Scoring Rubric

On the last page is the **problem-specific rubric**, to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on any problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Pre-Algebra PoWs. Please let me know if you have ideas for making them more useful.

~ Lillian and Suzanne <lillian@mathforum.org> <suzanne@mathforum.org>

Pre-Algebra Scoring Rubric for Suli's Sense

For each category, choose the level that best describes the student's work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	does none or one of the things listed under Practitioner	does two of the things listed under Practitioner	attempts to explain how Suli knows that the sum of the differences is 0 understands how to calculate the mean understands that there are seven data points, since it is a week	there is no Extra, and no way to be an Expert in this category for this problem
Strategy	does not have any ideas about how to solve the problem	has some ideas about how to solve the problem, but isn't quite there	has a strategy that relies on skill, not luck might construct a data set with a mean of 72 might work from the sum of the data (504 or seven 72s) might use the idea of zero pairs might graph their data	uses two separate strategies
Accuracy	has made many errors	makes a few errors that lead to an incorrect answer	work is accurate and contains no arithmetic mistakes uses appropriate language and units, such as degrees F	[not normally available for this category]
Communication				
Completeness	has written nothing that tells you how they found their answer	shows work without an explanation or explains everything without showing the numbers doesn't include enough information for another student to follow	attempts to explain all of the steps taken to solve the problem, which might include <ul style="list-style-type: none"> details of how they constructed a data set that would work demonstrating pairs of numbers that have a mean of 72 so that, if taken together, they will also have a mean of 72 	adds in useful extensions and further explanation of some ideas involved
Clarity	explanation is very difficult to read and follow	another student wouldn't be able to follow their explanation entirely long and written in one paragraph lots of spelling errors/typos	explains all of the steps mentioned in such a way that another student would understand makes an effort to check their formatting, spelling, and typing (a few errors are fine)	formats things exceptionally clearly answer is very readable and appealing
Reflection	<i>The items in the columns to the right are considered reflective, and could be in the solution or the comment they leave after viewing our answer:</i> does nothing reflective	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer does one reflective thing	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used does two reflective things	comments on and explains the ease or difficulty of the problem revises their answer and improves anything does three or more reflective things or an great job with two